Chapter 5
SPATIAL EQUILIBRIUM MODEL

Mihoko Shimamoto[1]

Contents

1.   Basic Concept of Spatial Equilibrium Model

Spatial Equilibrium Models can be defined as models which solve the simultaneous equilibria of plural regional markets under the assumption of existence of transportation costs between two regions. This complicated proposition can be arranged into a simpler style by applying the theorem that the solution of the competitive equilibrium is equal to the one of the maximization of social surplus (i..e. the total amount of producer surplus and consumer surplus) under perfect competitive market conditions. Samuelson (1952) indicated that a unique equilibrium solution could be found by the maximization of the total area under the excess demand curve in each region minus the total transportation costs of all shipments. The implication of his indication can be regarded as follows.

The two-variable case gives us an image of the principle. Figure 1. is the back-to-back diagram determining the equilibrium flow of exports of 2 regions (or two markets). D_1, S_1, D_2, S_2 are the demand and supply curves of these 2 countries. ES_1 and ES_2 indicate the excess supply curves.
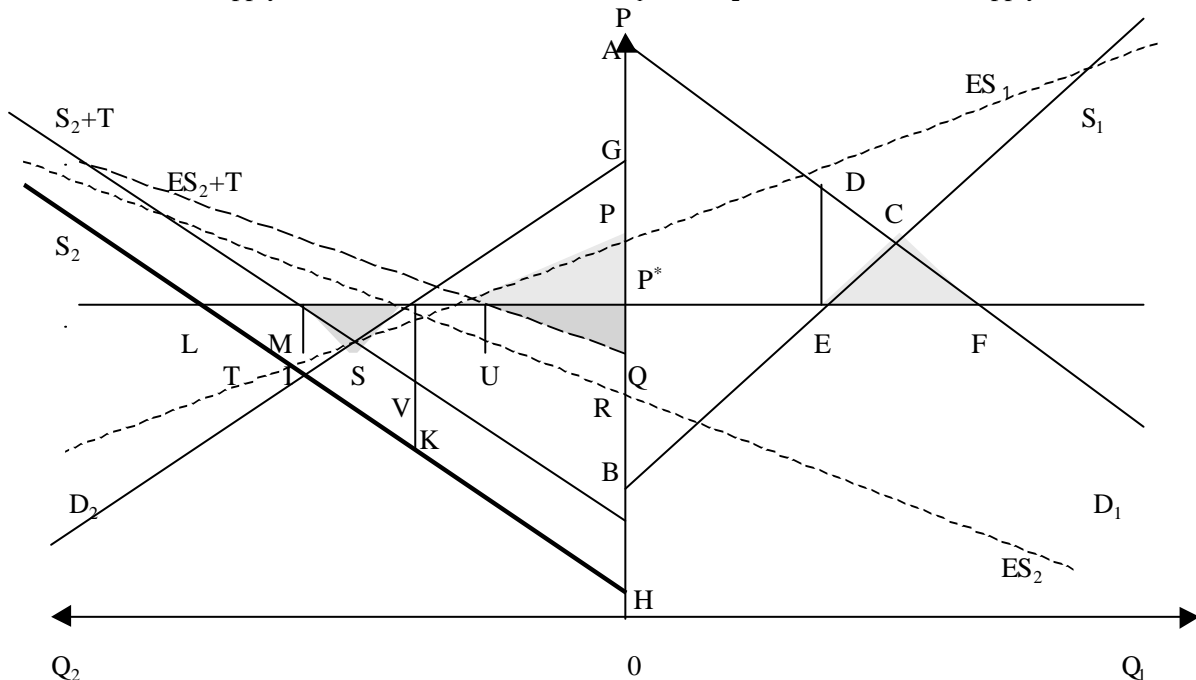

Figure 1. Simultaneous Equilibrium of 2 Markets

If the economies were closed in region 1 and 2, the market equilibrium of region 1 and 2 would have been C and I for each region. In the situation the social surplus of region 1 and 2 would

---

[1] Hosei University
[2] This section is completely based on ITTO (1993)

be ▵ABC and ▵GHI.

Now suppose that the economies are open. The transportation cost is assumed to be T (QR) dollars per unit. The export will start from region 2 to region 1 in this case. Equilibrium price will be at P^* because the excess supply of the exporter will be equivalent with the excess demand of the importer in this price level if the transportation cost is included into the marginal costs of the exporter. That means that EF is equal to MJ which indicates the distance of D_2 and S_2+T at the price of P^*.

The social surplus will increase by this trade. In region 1 the domestic supply is P^*E, so the social surplus is the area of ▵ABED. By the foreign goods the consumer surplus will increase by ▵DEF. Therefore the total area of the social surplus is ▵ABED+▵DEF in region 1. Comparing with the case of the closed economy, the social surplus of region 1 increased by ▵CEF. This area could be called the net increment of the social surplus by trade. It could be also indicated by ▵OPP* which is the description by the excess supply curve in region 1.

Next in region 2, if there were not any transportation costs, the same kind of explanation would be possible for the exporter. It means that Comparing with the case of the closed economy, the increment of social surplus is just equivalent with the area under the equilibrium price over the excess supply curve. Therefore the maximization of the total area under the excess supply curves means the maximization of the total net increment of the social surplus by trade (i.e. the maximization of total social surplus).

Now consider the case with the transportation cost T. Samuelson defined that the net social pay-off is the sum of the area over the excess supply curve (i.e. under the excess demand curve) minus the total transportation costs of all the shipment. It is indicated by ▵OPP*+ ▵OP*RU (i.e. the total of the area over the excess supply curves in region 1 and 2) – ▵OQRU (i.e. the transportation cost), which is equal to ▵OPQ. ▵OPP* is the net increment of the social surplus in region 1 from the trade as described before. As for region 2, it is rather complicated because the social surplus in the case of open economy is GHKVMJ. In the case of closed economy the social surplus is ▵GHI, so the net increment of the social surplus is ▵MJS- ▵SVKI. ▵MJS is equal to ▵OQP*.

Therefore the total net increment of the social surplus from trade is not equal to the ▵OPQ which is the net social pay-off defined by Samuelson. But they are very similar except the point that the area of total net increment of the social surplus from the trade has to be subtracted ▵SVKI from the net social pay-off.

Samuelson indicated that the maximization of the net social pay-off in plural markets has a unique solution of equilibrium. It is well known as the solution of spatial equilibrium in completely competitive markets.

2. Important Assumption of Spatial Equilibrium Model

The spatial equilibrium model which treats plural markets is not based on general equilibrium structure but on partial equilibrium analysis. Therefore the concept of consumer surplus or social surplus requires careful treatment when the analysis is about plural markets which are mutually related.

In order to describe the problem clearly, first consider about the case of two goods[3]. The

---

[3] Generally the problem of utility maximization can be written as;

Max U($\mathbf{x}$)

s.t. y $\mathbf{px'}=0$

where $\mathbf{x}=(x_1, x_2, \ldots, x_n)$ is a vector of commodities, $\mathbf{p}=(p_1, p_2, \ldots, p_n)$ is a vector of strictly positive prices, y>0 is income. The first order conditions for an interior solution to this problem are

$$\frac{\partial U(x)}{\partial x}$$

where is the Lagrange multiplier of the budget constraint. By solving these conditions the demand function of each goods and indirect utility function appear.

$x_1=x_1(\mathbf{p,y})$

demand curve of good 1 is $x_1(p_1,p_2{}^0,y)$, and that of good 2 is $x_2(p_1{}^0,p_2,y)$ at the start. Now the price of good 1 is suppose to change from p^o to $p_1{}^1$. The change in consumer surplus of good 1 is given by area A in Figure 2.1. The change in p_1 shifts the position of the demand curve of good 2 to the right (left) if good 1 and good 2 are complements (substitutes). However, if income and price of good 2 are fixed, the total change in consumer surplus is still given by area A in the figure.

Next , lower the price of good 2. The change in consumer surplus of this good must be evaluated given the fact that we have already reduced the price of good 1. Thus the relevant change in consumer surplus of good 2 is equal to area B+C in Figure 2.2. The total change of consumer surplus caused by the combined fall in p_1 and p_2 is equal to area A+B+C.

Assume that we instead lower p_2 before p_1. The change in consumer surplus of good 2 is now measured to the left of the demand curve drawn for p_1 $p_1{}^o$, i..e. it is equal to area B in Figure 2.2. As the price of good 2 is lowered, the demand curve for the good 1 may move leftward or rightward. In any case the change in the consumer surplus of good 1 must be evaluated to the left of the 'final' demand curve obtained for $p_2 = p_2{}^1$, is equal to area A+D in Figure 2.1.

In general the considered paths of price adjustment imput different gains or total consumer surpluses to the underlying unique change in utility, i.e. area A+B+C need not be equal to area B+A+D. In addition, it should be noted that these areas are obtained by considering just two out of possibly an infinite number of paths between initial and final prices since we could proceed by alternate, small changes in p_1 and p_2.

However if the cross-price effects are equal as follows,

$$\frac{\partial x_1}{\partial p_2} = \frac{\partial x_2}{\partial p_1}$$

the change in total consumer surplus will not depend on which procedure or path we take. It is called path independency.

The condition[4] of utility function which satisfy path independency of is homothetic or quasi-linear i.e. $U = u(x_1, x_2, \ldots\ldots, x_{n-1}) + a\, x_n$.
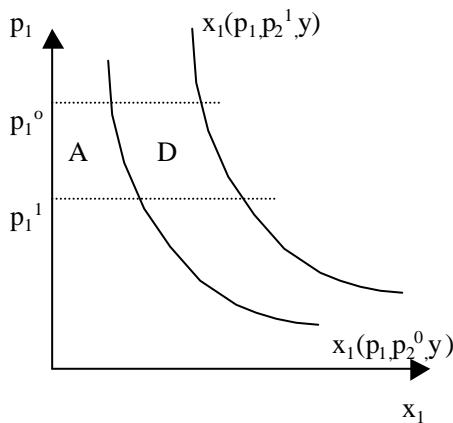

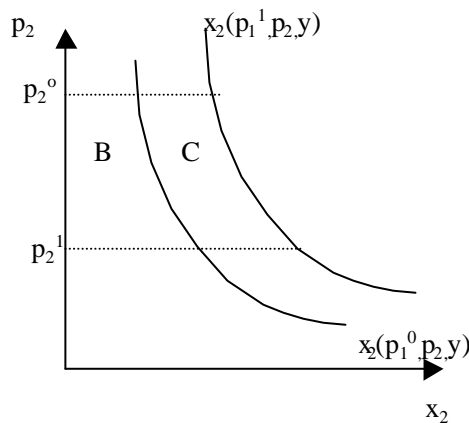Figure 2.1 Demand curve of good 1   Figure 2.2 Demand curve of good 2

3.   Framework of the Standard Spatial Equilibrium Model

Now we make the concept described in Section 1 a more concrete model. The standard one is the model used in the IIASA's Global Trade Model. The way to construct the model is as follows.

Consumer Sector (

$x_n = x_n(\mathbf{p},\mathbf{y})$

$U(x(\mathbf{p},\mathbf{y})) = V(\mathbf{p},\mathbf{y})$

[4] How the conditions were induced is in Johansson (1987) in detail.

Before stating the general structure of CSM, first consider the case of one product and a given demand function by employing Figure 3. Let p be the product price, q the product and q the product demand, and P(q) the inverse of the demand function. Given the price p, the demand q_d can be determined by maximizing over q the area GFCD.

$$\max_q \int_0^q P(q)dq - pq$$

On the basis of this individual market, the general structure of CSM will be described as follows,

$$\max_q$$

s.t.

where p is a vector of product prices, q is a vector of product demands, q_k, k=1,…..,K;   is the closed, convex, and nonempty consumption possibility set; and U   is a continuous and concave function defined over C, measuring the total benefit to the consumers. Therefore U(q) –   pq indicates a consumer surplus function.

Figure 3 CSM and PSM
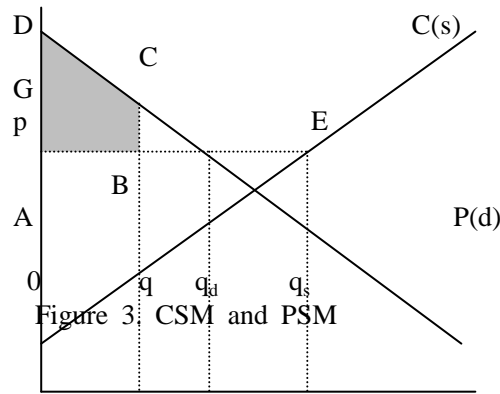
Production Sector

First derive a mathematical programming formulation of the PSM for the case of one product and given supply function by employing Figure 3. Let z be the production and C (z) the inverse of the supply function. Given the price p, the supply $z=q_s$ can be determined by maximizing over q the area ABFG, i.e. maximizing over z,

$$\max_z pz - \int_o^z C(z)dz$$

This problem is a specialization of the following problem, which we adopt as the general structure of the PSM. Given price vector p,

$$\max_z$$

s.t.

where z is a vector of product supplies, z_k, Z is the closed, convex, and nonempty production possibility set, and V is a continuous and convex costs measuring the producer's costs. Therefore we can call pz – V(z) the producer's surplus function.

Export-Import Sector

Consider the maximization of the profit from the trade with the other regions in each region. In region r the profit from the export is the residual of the import price from the exporter r to the importer s from which we subtract the domestic price in region r and the transportation costs. In region r the profit from the import is that the residual of the domestic price in the importer r from which we subtract the import price from exporter s. The import and export volumes will be obtained from the maximization of the sum of these trade profits in all the products and all the regions.

$$\max_{e_{rS},\, m_{rS}} \sum_{s,k} \qquad r \qquad r \quad r \qquad r \qquad r \qquad r \qquad r$$

$$\sum_{s,k} \qquad r \qquad r \qquad r \qquad r \qquad r$$

$$r \sum_{s} \qquad r \qquad r$$

s.t. $\qquad r \qquad r \qquad r$

$\qquad r \qquad$ …the import price in region s for products k from region r

$\qquad r \qquad$ …the vector of $\quad_r$

$\qquad$ …the vector of $\quad_r$

$\quad_r$…the vector of domestic prices $\qquad_r$ in region r

$\quad r \qquad$ …the vector of exports $\qquad_r \qquad$ from region r to region s

$\quad r \qquad$ …the vector of imports $\qquad_r \qquad$ to region r from region s

$\quad r \quad \cdots \quad r \quad \cdots \quad r$

$\qquad \cdots \quad r \quad \cdots \quad r$

$\quad r \qquad$ …the unit transportation cost from region r to region s for product k

$\quad r$…the closed, convex, and nonempty trade constraint set

Regional Models

The given price p in $\qquad$ and the one in $\qquad$ must be equivalent in the regional model. This price p must be also equivalent with the domestic price $\quad_r$ in $\qquad$. In these conditions the solution of the individual maximization problem of $\qquad$ and $\qquad$ under the given prices is equal to the solution of the maximization of the sum of the objective functions of CSM, PSM and TSM under the given $\qquad$ and $\quad_r$.

Now $\quad_r \quad_r \quad_r$ is the regional benefit function where:

$\quad r \quad r \quad r \quad r \quad r \qquad r$

The maximization problem of the sum of the objective functions of CSM, PSM and TSM in region r can be expressed as follows;

$$\max_{q_r,\, z_r\, e_{rS},\, m_{rS}} \quad r \quad r \quad r \sum_{s,k} \quad r^{*} \quad r \qquad r \qquad r \qquad r$$

$$r \quad r \quad r \sum_{s} \quad r \quad r$$

s.t. $\quad_r \quad_r \quad_r \quad_r \qquad_r$

$\qquad$ is a Cartesian product of $\qquad$ and

In the market equilibrium in each goods k, $\quad_r \quad_r$ is equal to $\qquad_r \quad_r$. Adding this constraint, price p_r becomes endogenous variable in the aggregated maximization problem in region r. It means the previous maximization problem is equal to the following one.

$$\max_{q_r,\, z_r\, e_{rS},\, m_{rS}} \quad r \quad r \quad r \sum_{s,k} \quad r \qquad r \qquad r \qquad r \quad r$$

s.t. $\quad r \quad r \sum_{s} \quad r \qquad r$

Global Model

The global model is the maximization problem of the aggregated function consisted of the

413

objective functions in the regional model for all the regions. Under the given price ;

$$\max_{q_r, z_r, e_{rS}, m_{rS}} \sum_r r \qquad r, \ r \qquad \sum_{rsk} r \qquad r$$

$$\sum_{rsk} r \qquad r \qquad r$$

$$\text{s.t.} \quad r \quad r \sum_s r \quad r \qquad \text{for all}$$

The import volume r of the product k from region s to region r is equal to the export volume r of the product k from region s to region r, therefore finally the global model can be indicated as follows.

$$\max_{q_r, z_r, e_{rS}, m_{rS}} \sum_r r \qquad r, \ r \qquad \sum_{rsk} r \qquad r$$

$$\text{s.t.} \quad r \qquad r \qquad \qquad \text{for all}$$
$$r \quad r \sum_s r \quad r \qquad \text{for all}$$

We can obtain all the volumes and prices of each product.

4. Review of Spatial Equilibrium Model[5]

     The tart of the use of spatial equilibrium concept in forest products sector dates back to the early 1960's. Employing a spatial allocation model, Holland and Judge (1963) studied the least cost flows of hardwood and softwood sawnwood for 11 demand regions and 18 supply regions in the United States. The spatial allocation model was based on the transportation problem in linear programming. As the objective was to minimize total transportation costs subject to constraints regarding demand and supply; prices were excluded from the model.

     Holley (1970) used a similar approach for examining sawnwood and plywood demand, supply and trade in the United States. Holley (1970) enhanced the work by Holland and Judge by including logging and manufacturing costs in the objective function. Using projections to the year 1975 from the base year 1965, he studied the shifts required to bring about the most efficient location of the industry.

     Holley et al. (1975) later extended the earlier work by using a linear program to model the least cost trade flows of eleven forest products in North America. This model was called ITM (Inter-Regional Trade Model). Timber availability and processing capacities offered constraints to the amount of products that could be consumed. Taking the future domestic consumption requirements as given, ITM simulated the future pattern of wood industry output and location. The objective function involved minimizing the overall cost of meeting customer demands.

     Inclusion of more explicit economic theory into trade modeling was possible with the development of reactive and separable programming which allowed non-linear functions to be approximated by division into a number of linear components and then solved using linear programming. In general terms, these models are referred to as spatial equilibrium models.

     The main difference between the earlier spatial allocation models and the spatial equilibrium models is that in the latter, supply and demand are expressed as functions, founded on economic theory, and not as fixed values. The other main difference is that the objective function is no longer to minimize costs but rather to maximize the surplus value of trade, or the sum of all consumer and producers surpluses.

     The work by Haynes et al. (1978) was among the first to use spatial equilibrium to model activity in the forest sector. The demand for softwood forest products in the United States was made a function of population, GNP and housing starts. Product prices were determined by substituting the

---

[5] This section totally depends on ITTO(1993)

equilibrium quantities consumed in each region into the demand function.

The Timber Assessment Market Model (TAMM), a spatial equilibrium model developed by Adams and Haynes (1980), is still used today to provide long-range projections of consumption, production, price and product flows for softwood lumber, plywood, and raw materials. The focus of the model is on the United States but it does include Canada. The model is capable of providing annual forecasts over a forty year period. Demand and supply are modeled using econometric relationships and the model is highly detailed in its specifications of production processes.

International trade modeling proliferated in the early 1980s. Some of the first work was by Buongiorno and Gilless (1982) who used a spatial equilibrium model to analyze the pulp and paper industry. Emphases were placed on the United States and Canada but the model also included Western Europe, Japan and the rest of the world.

Gilless and Buongiorno (1987) continued their efforts by designing PAPYRUS: A Model of the North American Pulp and Paper Industry, which was a price-endogenous linear programming model. Later, Zhang, Buongiorno and Ince (1992) refined PAPYRUS to handle any economic sector in a multi-commodity, multi-regional setting. Their model is now referred to as PELPS III (Price Endogenous Linear Programming System). The model provides forecasts of consumption, production, capacity, prices and trade within or among several regions or countries and for several commodities. This model is growing in acceptance because of its ability to be modified as more and better data arises and its operability on a micro-computer.

One of the most advanced trade models in terms of the number of regions (which were 43) and commodities (which were 10) is the Global Trade Model (GTM). This model was originally developed at the Institute of Applied Systems Analysis (Dykstra and Kallio, 1987) but has since been modified at the Center for International Trade in Forest Products (CINTRAFOR, for example Perez-Garcia 1993). The GTM has been applied to study several issues affecting the global forest products sector, including some recent work regarding tropical timber supplies (eg. LEEC, 1992)

The solution of the GTM requires a nonlinear programming optimizer (MINOS) developed at Stanford University and is reported to require between one and four hours of mini-computer time (VAX-780) to solve (Dykstra and Kallio, 1987) Its complexity and intensive computer requirements are the main weakness. The GTM has some technical weaknesses as well but they are common to other trade models.

But as there is an important advantage in models by spatial equilibrium which can include a large number of regions and commodities, the international organizations which are concerned with forest products, ITTO (ITTO(1993) and FAO (FAO(1997)), also constructed trade models of forest products using spatial equilibrium models.

References

Adams, D. M. and Haynes, R. W. (1980) 'The 1980 Softwood Timber Assessment Market Model: Structure, Projections, and Policy Simulations', *Forest Science* Monograph 22.

Buongiorno, J. and Gilless, J. K. (1982) *Concepts Used in a Regionalized Model of the Pulp and Paper Sector.* North American Conference on Forest Sector Models. Williamsburg, Va.

Dyktra and Kallio (1987) Global Forest Sector, NY: John Wiley and Sons.

FAO (1997) *FAO Provisional Outlook for Global Forest Products Consumption, Production and Trade to 2010*, Rome : FAO.

Gilless, J. K. and Buoniorno, J. (1987) PAPYRUS: A Model of the North American Pulp and Paper Industry, *Forest Science* Monograph 28.

Haynes, R. W., Holley, D. L. and King, R.A. (1978) 'A Recursive Spatial Equilibrium Model of the Softwood Timber Sector', *Technical Report No. 57. School of Forest Resources, North Carolina State University*.

Holland, I. I. and Judge, G. G. (1963) 'Estimated Interregional Flows of Hardwood and Softwood

Lumber', *Journal of Forestry* 61, pp.488-492.

Holley, D. L. (1970) 'Location of the Softwood Plywood and Lumber Industries: A Regional Programming Analysis', *Land Economics* 46 pp.127-137.

Holley, D. L., Haynes, R. W. and Kaiser, H. F. (1975) 'An Interregional Timber Model for Simulating Change in the Softwood Forest Economy', *School of Forest Resources North Carolina State University. Technical Report* No. 54.

ITTO (1993) Analysis of Macroeconomic Trends in the Supply and Demand of Sustainably Produced Tropical Timber from the Asia-Pacific Region Phase I, Yokohama: ITTO.

Johansson, Per-Olov (1987) *The Economic Theory and Measurement of Environmental Benefits*, NY: Cambridge University Press.

LEEC (1992) *The Economic Linkages between the International Trade in Tropical Timber and the Sustainable Management of Tropical Forests*, London: LEEC.

Perez-Garcia, J.M. (1993) *Global Forestry Impacts of Reducing Softwood Supplies from North America*, CINTRAFOR Working Paper 43.

Samuelson, P. A. (1952) 'Spatial Price Equilibrium and Linear Programming', American Economic Review 42, pp.283-303.

Zhang, D., Buongiorno, J. and Ince, P. J. (1992) *PELP III: A Micro-Computer Price Endogenous Linear Programming System for Economic Modeling.* Draft Research Paper. United States Department of Agriculture. Forest Service. Forest Products Laboratory.